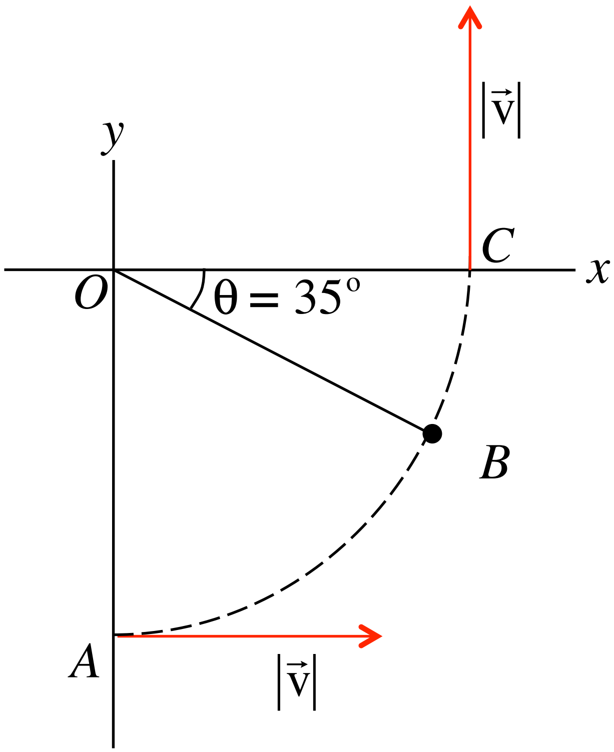
Problem 6.6

a.) Given that the vehicle is moving at a constant rate that takes it 235 meters in 36.0 seconds, what are its acceleration components (relative to the x and y-axes) when at Point B



Before we do anything, we need to determine the "constant velocity" magnitude. Using the definition of average velocity magnitude (which is the same as the speed if the body is moving with constant velocity magnitude), we can write:

$$|\vec{v}| = \frac{235 \text{ m}}{36.0 \text{ s}} = 6.53 \text{ m/s}$$

We will also need the radius, which can be determined using the circumference relationship, or:

$$\frac{1}{4}C = \frac{1}{4}(2\pi R) = (235 \text{ m})$$

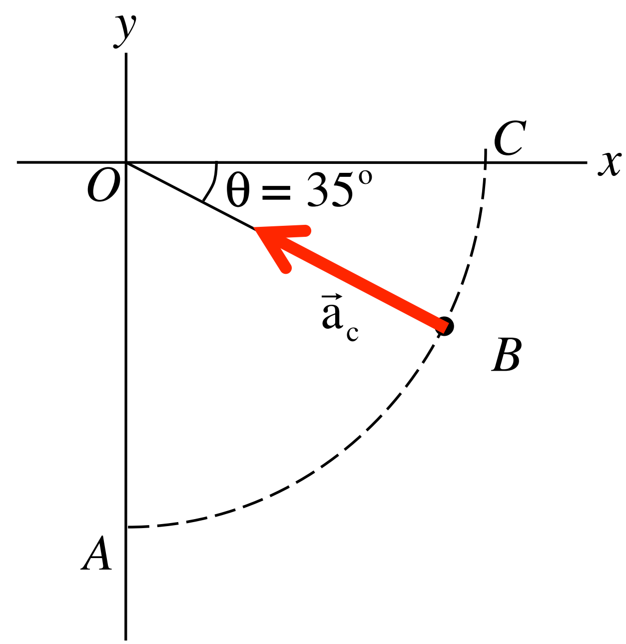
$$\Rightarrow \quad R = 150. \text{ m}$$

The only acceleration that is happening is in the radial direction. It's magnitude is:

$$a_c = \frac{v^2}{R}$$

$$= \frac{(6.53\ \text{m/s})^2}{(150\ \text{m})}$$

$$= .285\ \text{m/s}^2$$

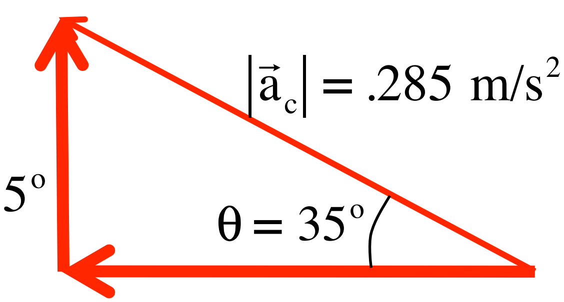


That acceleration is depicted on the sketch. Breaking it into x/y components as shown on the triangle shown yields:

$$|\vec{a}_y| = |\vec{a}_c| \sin 35^\circ$$

$$= (.285\ \text{m/s}^2) \sin 35^\circ$$

$$= (.163\ \text{m/s}^2)$$



$$|\vec{a}_x| = |\vec{a}_c| \cos 35^\circ$$

$$= (.285\ \text{m/s}^2) \cos 35^\circ$$

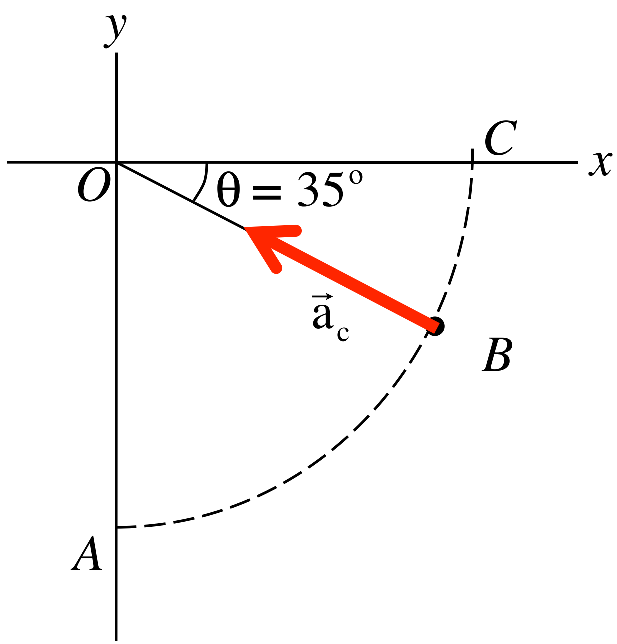
$$= (.233\ \text{m/s}^2)$$

OR

$$\vec{a} = (.233\ \text{m/s}^2)(-\hat{i}) + (.163\ \text{m/s}^2)\hat{j}$$

$$= -(.233\ \text{m/s}^2)(\hat{i}) + (.163\ \text{m/s}^2)\hat{j}$$

Just as a minor note, this is really an exercise in futility. There is an acceleration in the radial, center-seeking, centripetal direction (a unit vector in that direction is symbolized as $\hat{r}$). We found that to be .285 m/s/s. There is no acceleration in the direction perpendicular to that, which is called the "tangential direction" (its unit vector is symbolized as $\hat{\theta}$), This is evident as we were told the velocity magnitude is constant, meaning there is no acceleration along that line of motion. If the authors of the book had wanted to give you a break and had just asked for the acceleration at the point indicated (without demanding it be relative to an x/y coordinate axis), you could have easily accommodated by writing:

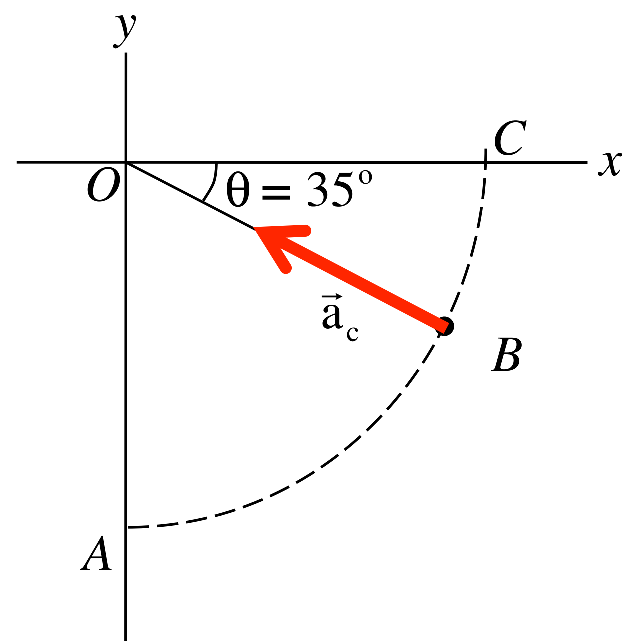


$$\begin{aligned}\vec{a} &= \left(.285\ \text{m/s}^2\right)\left(-\hat{r}\right) + \left(0\right)\hat{\theta} \\ &= -\left(.285\ \text{m/s}^2\right)\hat{r}\end{aligned}$$

and you'd have been done. No trig, no hassle, just easy, functional, appropriate notation!

b.) The average speed is just the constant velocity magnitude, or 6.53 m/s.

c.) The average acceleration during the motion is equal to the change in velocity divided by the time involved. Executing that operation, we get:



$$\begin{aligned}\vec{a}_{avg} &= \frac{\Delta\vec{v}}{\Delta t} \\ &= \frac{\vec{v}_{final} - \vec{v}_{initial}}{\Delta t} \\ &= \frac{(6.53\text{ m/s})\hat{j} - (6.53\text{ m/s})\hat{i}}{36.0\text{ s}} \\ &= \frac{-(6.53\text{ m/s})\hat{i} + (6.53\text{ m/s})\hat{j}}{36.0\text{ s}} \\ &= -(.181\text{ m/s})\hat{i} + (.181\text{ m/s})\hat{j}\end{aligned}$$

Without showing the math, note that this in polar notation is equal to $.256\measuredangle 135^{\circ}$.